

MATHEMATICS

(Three hours)

(Candidates are allowed additional 15 minutes for only reading the paper.

They must NOT start writing during this time.)

Section A - Answer **Question 1** (compulsory) and five other questions.

Section B and Section C - Answer two questions from either Section B or Section C.

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [].

Mathematical tables and graph papers are provided.

Slide rule may be used.

SECTION A

Question 1

- (i) Solve for x and y if $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2 \begin{pmatrix} 2x \\ 3y \end{pmatrix} = 3 \begin{pmatrix} 7 \\ -3 \end{pmatrix}$ [3]
- (ii) Prove that $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3) = 15$ [3]
- (iii) Find the equation of the hyperbola whose Transverse and Conjugate axes are the x and y axes respectively, given that the length of conjugate axis is 5 and distance between the foci is 13. [3]
- (iv) From the equations of the two regression lines, $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$, find: [3]
- (a) Mean of x and y .
- (b) Regression coefficients.
- (c) Coefficient of correlation.
- (v) Evaluate: $\int e^x (\tan x + \log \sec x) dx$ [3]
- (vi) Evaluate: $\lim_{x \rightarrow \frac{\pi}{2}} (\cos x \cdot \log \tan x)$ [3]
- (vii) Find the locus of the complex number, $Z = x + iy$ [3]

$$\text{given } \left| \frac{x + iy - 2i}{x + iy + 2i} \right| = \sqrt{2}$$

This Paper consists of 6 printed pages.

(viii) Evaluate: $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx$ [3]

(ix) Three persons A, B and C shoot to hit a target. If in trials, A hits the target 4 times in 5 shots, B hits 3 times in 4 shots and C hits 2 times in 3 trials. Find the probability that: [3]

(a) Exactly two persons hit the target.

(b) At least two persons hit the target.

(x) Solve the differential equation: [3]

$$(xy^2 + x)dx + (x^2y + y)dy = 0$$

Question 2

(a) Using properties of determinants, prove that: [5]

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

(b) Find the product of the matrices A and B where: [5]

$$A = \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \text{ Hence, solve the following equations by}$$

matrix method:

$$x + y + 2z = 1$$

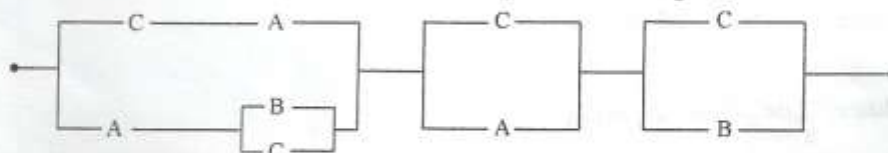
$$3x + 2y + z = 7$$

$$2x + y + 3z = 2$$

Question 3

(a) Prove that: $\cos^{-1} \frac{63}{65} + 2 \tan^{-1} \frac{1}{5} = \sin^{-1} \frac{3}{5}$ [5]

(b) (i) Write the Boolean expression corresponding to the circuit given below: [5]



(ii) Simplify the expression using laws of Boolean Algebra and construct the simplified circuit.

Question 4

- (a) Verify Rolle's theorem for the function:

[5]

$$f(x) = \log \left\{ \frac{x^2 + ab}{(a+b)x} \right\} \text{ in the interval } [a, b] \text{ where, } 0 \notin [a, b].$$

- (b) Find the equation of the ellipse with its centre at (4, -1) focus at (1, -1) and given that it passes through (8, 0).

[5]

Question 5

- (a) If,
- $e^y(x+1) = 1$
- , then show that:

[5]

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx} \right)^2$$

- (b) A printed page is to have a total area of 80 sq. cm with a margin of 1 cm at the top and on each side and a margin of 1.5 cm at the bottom. What should be the dimensions of the page so that the printed area will be maximum?

[5]

Question 6

- (a) Evaluate:
- $\int \frac{dx}{x \{ 6(\log x)^2 + 7 \log x + 2 \}}$

[5]

- (b) Find the area of the region bounded by the curve
- $x = 4y - y^2$
- and the y-axis.

[5]

Question 7

- (a) Ten candidates received percentage marks in two subjects as follows:

[5]

Candidate	A	B	C	D	E	F	G	H	I	J
Mathematics Marks	80	88	76	74	68	65	40	43	40	80
Statistics Marks	72	84	90	66	54	50	54	38	30	43

Calculate Spearman's rank correlation coefficient and interpret your result.

- (b) The following results were obtained with respect to two variables x and y : [5]

$$\sum x = 30, \sum y = 42, \sum xy = 199, \sum x^2 = 184, \sum y^2 = 318, n = 6$$

Find the following:

- The regression coefficients.
- Correlation coefficient between x and y .
- Regression equation of y on x .
- The likely value of y when $x = 10$.

Question 8

- A bag contains 8 red and 5 white balls. Two successive draws of 3 balls are made at random from the bag without replacements. Find the probability that the first draw yields 3 white balls and the second draw 3 red balls. [5]
- A box contains 30 bolts and 40 nuts. Half of the bolts and half of the nuts are rusted. If two items are drawn at random from the box, what is the probability that either both are rusted or both are bolts? [5]

Question 9

- (a) Using De Moivre's theorem prove that: [5]

$$\left(\frac{1 + \cos \theta + i \sin \theta}{1 + \cos \theta - i \sin \theta} \right)^n = \cos n\theta + i \sin n\theta, \text{ where } i = \sqrt{-1}$$

- (b) Solve the differential equation: [5]

$$\frac{dy}{dx} - 3y \cot x = \sin 2x, \text{ given } y = 2, \text{ when } x = \frac{\pi}{2}$$

SECTION B

Question 10

- (a) For any three vectors $\vec{a}, \vec{b}, \vec{c}$ prove: [5]

$$[\vec{a} - \vec{b} \quad \vec{b} - \vec{c} \quad \vec{c} - \vec{a}] = 0$$

- (b) In any triangle ABC, prove by vector method: [5]

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Question 11

- (a) Find the shortest distance between the lines:

[5]

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \text{ and}$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{5-z}{5}$$

- (b) Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z - 5 = 0$ and $3x - 2y - z + 1 = 0$ and cutting off equal intercepts on the x and z axes.

[5]

Question 12

- (a) In a class of 75 students, 15 are above average, 45 are average and the rest below average achievers. The probability that an above average achieving student fails is 0.005, that an average achieving student fails is 0.05 and the probability of a below average achieving student failing is 0.15. If a student is known to have passed, what is the probability that he is a below average achiever?

[5]

- (b) The probability that a bulb produced by a factory will fuse in 100 days of use is 0.05. Find the probability that out of 5 such bulbs, after 100 days of use:

[5]

- (i) None fuse.
- (ii) Not more than one fuses.
- (iii) More than one fuses.
- (iv) At least one fuses.

SECTION C**Question 13**

- (a) Two tailors P and Q earn Rs.150 and Rs.200 per day respectively. P can stitch 6 shirts and 4 trousers a day, while Q can stitch 10 shirts and 4 trousers per day. How many days should each work to produce at least 60 shirts and 32 trousers at minimum labour cost?
- (b) A machine costs Rs.97,000 and its effective life is estimated to be 12 years. If scrap realises Rs.2,000 only, what amount should be retained out of profits at the end of each year to accumulate at compound interest of 5% per annum in order to buy a new machine after 12 years? (use $1.05^{12} = 1.769$)

[5]

[5]

Question 14

- (a) A bill of Rs.1,000 drawn on 7th May, 2011 for six months was discounted on 29th August, 2011 for cash payment of Rs.988. Find the rate of interest charged by the bank. [5]
- (b) If total cost function is given by $C = a + bx + cx^2$, where x is the quantity of output. Show that: [5]

$$\frac{d}{dx}(AC) = \frac{1}{x}(MC - AC), \text{ where } MC \text{ is the marginal cost and } AC \text{ is the average cost.}$$

Question 15

- (a) Find the consumer price index number for the year 2010 using year 2000 as the base year by using method of weighted aggregates: [5]

Commodity	A	B	C	D	E
Year 2000 price (in Rs. per unit)	16	40	0.50	5.12	2
Year 2010 price (in Rs. per unit)	20	60	0.50	6.25	1.50
Weights	40	25	5	20	10

- (b) Calculate the 5 yearly moving averages of the number of students in a College from the following data and plot them on a graph paper: [5]

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Number	332	317	357	392	402	405	510	427	405	438