MATHEMATICS

(Two hours and a half)

SECTION A (40 Marks)

*Attempt **all** questions from this Section.*

Question 1

(a)   Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by $x - 2$, leaves a remainder 7.   [3]

(b)   Given A = $\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = 9A + mI$. Find m.   [4]

(c)   The mean of following numbers is 68. Find the value of 'x'.   [3]

45, 52, 60, x, 69, 70, 26, 81 and 94.

Hence estimate the median.

Question 2

(a)　The slope of a line joining P(6, k) and Q(1-3k, 3) is $\frac{1}{2}$. Find　　　　[3]

　　(i)　k

　　(ii)　Midpoint of PQ, using the value of 'k' found in (i).

(b)　Without using trigonometrical tables, evaluate:　　　　[4]

$$cosec^2 57° - tan^2 33° + cos44°cosec46° - \sqrt{2}cos45° - tan^2 60°$$

(c)　A certain number of metallic cones, each of radius 2 cm and height 3 cm are melted　　　[3]
and recast into a solid sphere of radius 6cm. Find the number of cones.

Question 3

(a)　Solve the following inequation, write the solution set and represent it on the number　　[3]
line.

$$-3(x - 7) \geq 15 - 7x > \frac{x + 1}{3}, x \in R$$

(b)　In the figure given below, AD is a diameter. O is the centre of the circle.　AD is　　[4]
parallel to BC and $\angle$ CBD = 32⁰. Find:

　　(i)　$\angle$OBD

　　(ii)　$\angle$AOB

　　(iii)　$\angle$BED.
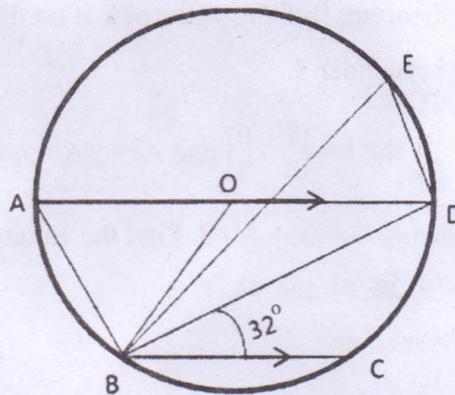
(c)　If $(3a + 2b):(5a + 3b) = 18:29$. Find $a : b$.　　　　[3]

Question 4

(a) A game of numbers has cards marked with 11, 12, 13, ..., 40. A card is drawn at random. Find the Probability that the number on the card drawn is: [3]

    (i) A perfect square

    (ii) Divisible by 7

(b) Use graph paper for this question.   [4]

    (Take 2 cm = 1 unit along both x and y axis.)

    Plot the points O (0, 0), A (–4, 4), B (–3, 0) and C (0, –3)

    (i) Reflect points A and B on the y axis and name them A' and B' respectively. Write down their coordinates.

    (ii) Name the figure OABCB'A'.

    (iii) State the line of symmetry of this figure.

(c) Mr. Lalit invested ₹5000 at a certain rate of interest, compounded annually for two years. At the end of first year it amounts to ₹5325. Calculate   [3]

    (i) The rate of interest.

    (ii) The amount at the end of second year, to the nearest rupee.

SECTION B (40 Marks)

*Attempt any **four** questions from this Section*

Question 5

(a) Solve the quadratic equation $x^2 - 3(x + 3) = 0$; Give your answer correct to two significant figures.   [3]

(b)   A page from the savings bank account of Mrs. Ravi is given below.   [4]

Date	Particulars	Withdrawal (₹)	Deposit (₹)	Balance (₹)
April 3rd 2006	B/F			6000
April 7th	By cash		2300	8300
April 15th	By cheque		3500	11800
May 20th	To self	4200		7600
June 10th	By cash		5800	13400
June 15th	To self	3100		10300
August 13th	By cheque		1000	11300
August 25th	To self	7400		3900
September 6th 2006	By cash		2000	5900

She closed the account on 30th September, 2006. Calculate the interest Mrs. Ravi earned at the end of 30th September, 2006 at 4.5% per annum interest. Hence, find the amount she receives on closing the account.

(c)   In what time will Rs.1500 yield Rs.1996.50 as compound interest at 10% per annum compounded annually?   [3]

Question 6

(a)   Construct a regular hexagon of side 5 cm. Hence construct all its lines of symmetry and name them.   [3]
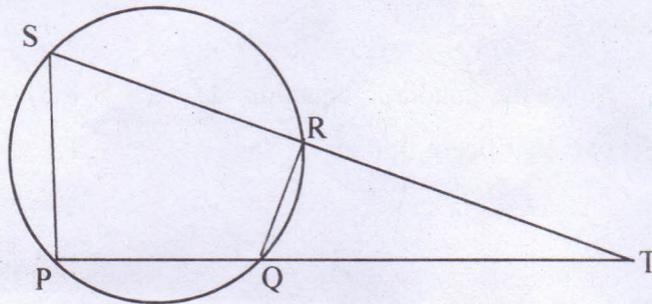
(b)   In the given figure PQRS is a cyclic quadrilateral PQ and SR produced meet at T.   [4]

(i)   Prove $\Delta TPS \sim \Delta TRQ$.

(ii) Find SP if TP = 18cm, RQ = 4 cm and TR = 6cm.

(iii) Find area of quadrilateral PQRS if area of $\Delta PTS = 27$ cm^2.

(c)   Given matrix $A = \begin{bmatrix} 4\sin 30° & \cos 0° \\ \cos 0° & 4\sin 30° \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ [3]

If AX =B

(i)   Write the order of matrix X.

(ii) Find the matrix 'X'.

Question 7

(a)   An aeroplane at an altitude of 1500 metres finds that two ships are sailing towards   [4]
it in the same direction. The angles of depression as observed from the aeroplane
are 45° and 30° respectively. Find the distance between the two ships.

(b)   The table shows the distribution of the scores obtained by 160 shooters in a   [6]
shooting competition. Use a graph sheet and draw an ogive for the distribution.
(Take 2cm = 10 scores on the X axis and 2cm = 20 shooters on the Y-axis).

Scores	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of shooters	9	13	20	26	30	22	15	10	8	7

Use your graph to estimate the following:

(i)   The median.

(ii) The interquartile range.

(iii) The number of shooters who obtained a score of more than 85%.

Question 8

(a)   If $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$ show that $\dfrac{x^3}{a^3} + \dfrac{y^3}{b^3} + \dfrac{z^3}{c^3} = \dfrac{3xyz}{abc}$ [3]

(b)   Draw a line AB = 5 cm. Mark a point C on AB such that AC = 3 cm. Using a ruler   [4]
and a compass only, construct:

(i)   A circle of radius 2.5 cm, passing through A and C.

(ii) Construct two tangents to the circle from the external point B. Measure and
record the length of the tangents.

(c) A line AB meets X-axis at A and Y- axis at B. P(4, -1) divides AB in the ratio 1:2. [3]

(i) Find the coordinates of A and B.

(ii) Find the equation of the line through P and perpendicular to AB.
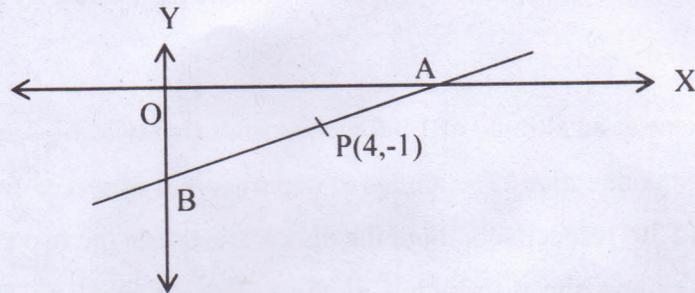
Question 9

(a) A dealer buys an article at a discount of 30% from the wholesaler, the marked price [3]
being ₹6,000. The dealer sells it to a shopkeeper at a discount of 10% on the marked
price. If the rate of VAT is 6%, find

(i) The price paid by the shopkeeper including the tax.

(ii) The VAT paid by the dealer.

(b) The given figure represents a kite with a circular and a semicircular motifs stuck [4]
on it. The radius of circle is 2.5 cm and the semicircle is 2 cm. If diagonals AC and
BD are of lengths 12 cm and 8 cm respectively, find the area of the:

(i) shaded part. Give your answer correct to the nearest whole number.

(ii) unshaded part.
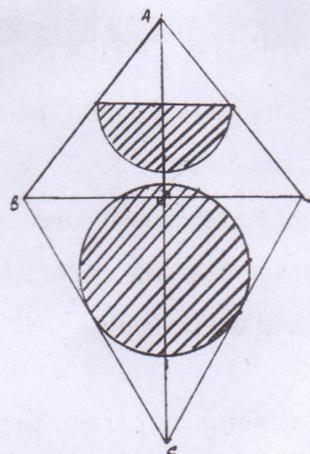
(c)   A model of a ship is made to a scale 1 : 300   [3]

   (i)   The length of the model of the ship is 2 m. Calculate the length of the ship.

   (ii) The area of the deck ship is 180, 000 m². Calculate the area of the deck of the model.

   (iii) The volume of the model is 6.5 m³. Calculate the volume of the ship.

Question 10

(a)   Mohan has a recurring deposit account in a bank for 2 years at 6 % p.a. simple   [3]
      interest. If he gets ₹ 1200 as interest at the time of maturity, find:

   (i)   the monthly instalment

   (ii) the amount of maturity.

(b)   The histogram below represents the scores obtained by 25 students in a   [4]
      Mathematics mental test. Use the data to:

   (i)   Frame a frequency distribution table.

   (ii) To calculate mean.

   (iii) To determine the Modal class.
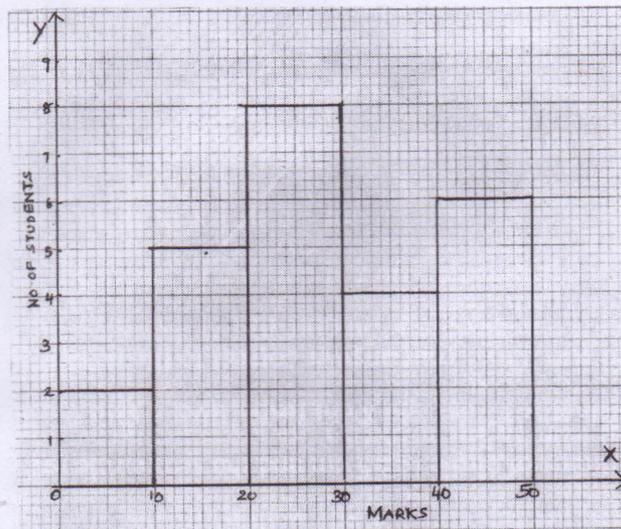
(c)   A bus covers a distance of 240 km at a uniform speed. Due to heavy rain its speed   [3]
      gets reduced by 10 km/h and as such it takes two hrs longer to cover the total
      distance. Assuming the uniform speed to be 'x' km/h, form an equation and solve
      it to evaluate 'x'.

Question 11

(a) Prove that $\frac{\cos A}{1+\sin A} + \tan A = \sec A$. [3]

(b) Use ruler and compasses only for the following question. All construction lines and arcs must be clearly shown. [4]

(i) Construct a $\triangle$ ABC in which BC= 6.5 cm, $\angle$ABC = 60°, AB= 5cm .

(ii) Construct the locus of points at a distance of 3.5 cm from A.

(iii) Construct the locus of points equidistant from AC and BC.

(iv) Mark 2 points X and Y which are at a distance of 3.5cm from A and also equidistant from AC and BC. Measure XY.

(c) Ashok invested ₹ 26,400 on 12%, ₹25 shares of a company. If he receives a dividend of ₹2,475, find the: [3]

(i) number of shares he bought

(ii) Market value of each share